obtain that $\lim_{n\to\infty} A_n = f(0)$ and $\lim_{n\to\infty} \ln G_n = \ln f(0)$. By the squeeze principle $\lim_{n\to\infty} U_n = f(0)$.

*Also solved by DIMITRIOS KOUKAKIS, Kato Apostoloi, Greece; PAOLO PERFETTI, Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, Rome, Italy; JOEL SCHLOSBERG, Bayside, NY, USA; and the proposer. Schlosberg used the harmonic mean instead of the geometric mean.*

**3686**. [2011 : 456, 458] *Proposed by Michel Bataille, Rouen, France.*

Let $a$, $b$, and $c$ be real numbers such that $abc = 1$. Show that

$$\left(a - \frac{1}{a} + b - \frac{1}{b} + c - \frac{1}{c}\right)^2 \leq 2\left(a + \frac{1}{a}\right)\left(b + \frac{1}{b}\right)\left(c + \frac{1}{c}\right).$$

*I. Solution by Arkady Alt, San Jose, CA, USA; and Titu Zvonaru, Cománeşti, Romania(independently).*

Let $x = a+b+c$ and $y = ab+bc+ca$. Since $abc = 1$, the difference between the two sides of the inequality is

$$\begin{aligned}
&2(a^2+1)(b^2+1)(c^2+1) - (a+b+c-bc-ca-ab)^2\\
&\quad = 2(2 + a^2b^2 + b^2c^2 + c^2a^2 + a^2 + b^2 + c^2) - (a+b+c-ab-bc-ca)^2\\
&\quad = 2(2 + y^2 - 2x + x^2 - 2y) - (x-y)^2 = (x^2 + y^2 + 2xy - 4x - 4y + 4)\\
&\quad = (x+y-2)^2 \geq 0.
\end{aligned}$$

Equality occurs if and only if $a+b+c+ab+bc+ca-2 = 0$.

*II. Solution by Šefket Arslanagić, University of Sarajevo, Sarajevo, Bosnia and Herzegovina; Oliver Geupel, Brühl, NRW, Germany; Kee-Wai Lau, Hong Kong, China; Salem Malikić, student, Simon Fraser University, Burnaby, BC; and Phil McCartney, Northern Kentucky University, Highland Heights, KY, USA (independently).*

Let

$$f(a,b) = 2(a^2+1)(b^2+1)(a^2b^2+1) - (a^2b + ab^2 + 1 - a - b - a^2b^2)^2.$$

Replacing $c$ by $1/ab$, we find that the difference of the two sides of the inequality is

$$\begin{aligned}
a^{-2}b^{-2}f(a,b) &= a^{-2}b^{-2}(1 + a + b - 2ab + a^2b + ab^2 + a^2b^2)^2\\
&= (c + ac + bc - 2 + a + b + ab)^2 \geq 0,
\end{aligned}$$

from which the result follows, with the foregoing condition for equality.

*III. Solution by AN-anduud Problem Solving Group, Ulaanbaatar, Mongolia; and Paolo Perfetti, Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, Rome, Italy(independently).*